MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

EEM2036 – ENGINEERING MATHEMATICS III

(RE/TE/ME)

14 OCTOBER 2019
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of six pages including cover page with four questions only.

2. Attempt **ALL** questions. The distribution of the marks for each question is given.

3. Please write down all your answers in the answer Booklet provided. Show all relevant steps to obtain maximum marks.

Question 1

a) Let $A=\begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 1\\ 2 & 0 & -3\end{bmatrix}$. Find A^{-1}.

[6 marks]

b) Given that $B=\begin{bmatrix}3 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 3\end{bmatrix}$.

i) Find the eigenvalues of B.

ii) Given that the eigenvectors for matrix B are $\begin{bmatrix}1\\2\\1\end{bmatrix}$, $\begin{bmatrix}-1\\0\\1\end{bmatrix}$ and $\begin{bmatrix}1\\-1\\1\end{bmatrix}$ corresponding to eigenvalues 1, 3 and 4 respectively. Find the matrix P and the diagonal matrix D such that matrix B can be diagonalized via $P^{-1}BP=D$.

iii) Given that the differential equations modelling a certain control system are

$$x_1' = 3x_1 - x_2$$
$$x_2' = -x_1 + 2x_2 - x_3$$
$$x_3' = -x_2 + 3x_3$$

Using the solutions from (ii) above, find the general solution for x_1, x_2 and x_3.

[5+4+10 marks]

Continued...

Question 2

a) Find the volume, V of the solid that is bounded above by the plane $z = x + 12$ and below by the region enclosed the parabola $y = 5 - x^2$ and the line $4x + y = 0$ in the $xy-$plane.

[7 marks]

b) Evaluate the integral $\int_0^1 \int_x^{\sqrt{2-x^2}} 3x + y \, dy \, dx$ by changing to polar coordinate.

[9 marks]

c) Using cylindrical coordinates, find the volume of the solid bounded by the paraboloid $z = 2 - x^2 - y^2$ and the conic surface $z = \sqrt{x^2 + y^2}$.

[9 marks]

Continued...

Question 3

(a) Evaluate $\iint_{\sigma} \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}}$ to find the flux of

$$\vec{\mathbf{F}}(x,y,z) = -\frac{2}{3}\hat{\mathbf{i}} - y\hat{\mathbf{j}} + \ln(x^2+y^2)\hat{\mathbf{k}}$$

where σ is part of the surface $z = y^2 + x^3$ bounded between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ in the first octant. Assume positive orientation.

[11 marks]

(b) Suppose a vector field is given by

$$\vec{\mathbf{F}}(x,y,z) = y^2\hat{\mathbf{i}} + z^2\hat{\mathbf{j}} + x^2\hat{\mathbf{k}}$$

(i) Find curl of $\vec{\mathbf{F}}$, i.e. $\nabla \times \vec{\mathbf{F}}$.

(ii) Hence, use Stokes' Theorem to evaluate $\oint_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where C is the closed path at the base of the cone $z = \sqrt{x^2 + y^2}$ cut out by $x = -1$, $x = 1$, $y = -1$ and $y = 1$. Assume positive orientation.

[3+11 marks]

Continued...

Question 4

(a) Approximate $\int_3^4 x\ln(x)dx$ using composite Simpson's rule with step size $h = 0.25$.

[8 marks]

(b) Compute the first 2 iterations of Newton-Raphson's method starting with $x_0 = 3$ to approximate $x\tan(x) = 0$.

[8 marks]

(c) Solve the initial value problem by using the Euler's method with h=0.1

$$y' = y\ln(x), \quad y(2) = 4 \quad \text{for} \quad 2 \le x \le 2.2$$

If the exact solution is $y = x^x e^{-x+2}$, calculate the absolute error for $y(2.2)$.

[9 marks]

End of Paper

Appendix

1. Composite Simpson's rule (m intervals)

$$\int_a^b f(x)dx \approx \frac{h}{3}\left(f(a) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + 2\sum_{j=1}^{(n/2)-1} f(x_{2j}) + f(b) \right)$$

with $n = 2m$, $h = \frac{b-a}{n}$ and error term $\left|\frac{(b-a)h^4}{180} f^{(4)}(\xi)\right|$ for some $\xi \in [a,b]$.

2. Newton-Raphson's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

3. Euler's method

$$y_{i+1} = y_i + hf(x_i, y_i)$$